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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

A REVISION OF THE ATOMIC WEIGHT OF STRONTIUM.

SECOND PAPER. — THE ANALYSIS OF STRONTIC CHLORIDE.

By Theodore William Richards.

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Presented January 11, 1905. Received January 7, 1905.

About ten years ago the first paper from this laboratory upon the subject of the atomic weight of strontium * showed that the usually accepted value 87.5 was probably about 0.2 per cent too low. This result was not a surprise, for the error had been predicted, upon the basis of a discussion of the faulty methods used by previous experimenters. The new results seemed to be trustworthy, and to represent, as well as a single series of experiments could, the true value of the constant sought.

The determination of an atomic weight by a single method is nevertheless unsatisfactory, no matter how admirable that method may be. Of course the verdict of a good method and a series of good analyses is much reinforced by an experimental proof of the flaws in other discrepant data; but the greatest of certainty is to be obtained only when the subject has been approached from several wholly different points of view. With this idea in mind, therefore, the analysis of strontic chloride was undertaken in order to confirm or disprove, as the case might be, the results of the analysis of strontic bromide described in 1894. The investigation was made in 1895–6, but has remained unpublished because of an incomprehensible discrepancy, now explained.

The balances and weights employed were those used in previous work, to which the reader is referred for all details. All weights were reduced to the vacuum standard as usual. The specific gravity of strontic chloride taken for this purpose was that given by Schroeder, 3.05, which was assumed to be correct, because his determination for baric chloride under similar conditions has often been confirmed. All the care taken in previous work at this laboratory was exercised in this case; it is unnecessary to repeat the several details.

The sample of strontic chloride used for analysis was made from a portion of the strontic carbonate remaining from the third preparation

* These Proceedings, **30**, 369 (1894).

of strontic bromide,* and was therefore no less pure than this specimen. The carbonate had been prepared in the following manner. The so-called "pure" strontic chloride of commerce was dissolved in water, treated with ammonic hydrate and a little carbonate, and filtered from the precipitate containing iron, aluminum, and so forth. To the filtrate was added an excess of sulphuric acid, and the precipitated strontic sulphate was thoroughly washed with dilute sulphuric acid and then with pure water, in the hope of freeing it from magnesium and calcium. When the wash water became neutral to methyl orange the precipitate was treated with enough ammonic carbonate solution to convert about half of it into carbonate, and the mixed precipitate was then washed with water by decantation until only a very small constant trace of sulphuric acid (due to strontic sulphate) was found in the decantate. The carbonate was then decomposed by pure hydrochloric acid, and the solution was allowed to stand in a glass flask for nine months over the undecomposed sulphate, with occasional shaking.† Thus all but a minute trace of the barium was eliminated. The strontic chloride was decanted, the sulphate washed once with water, and the filtered decanted liquid was evaporated in a platinum dish until most of the free hydrochloric acid had been expelled. The dissolved residue was neutralized with ammonia, shaken with a little ammonic carbonate, and then filtered. To the greatly diluted filtrate was added an excess of pure ammonic carbonate, and the precipitate was washed until the wash water was free from chlorine. The strontic carbonate was dissolved in nitric acid which had been twice distilled in platinum, and the nitrate was crystallized twice successively in a platinum dish. Each quantity of crystals was washed with small quantities of water and three or four additions of alcohol. The first mother liquor, upon being fractionally precipitated by means of alcohol, showed distinct traces of calcium in the extreme solution; thus Barthe and Falière's method was not capable of freeing the substance wholly from calcium. The second nitrate mother liquor showed no trace of calcium upon the most careful spectrometric scrutiny, although the test is one of very great sensitiveness; therefore the crystals could have contained none.‡ This effectual elimination of calcium was due no doubt to the difference in crystalline form between the calcium and strontium salts.

* These Proceedings, **30**, 377 (1894).

† Barthe and Falières, Journ. Chem. Soc., Abs. 1892, p. 1277. Bull. Soc. Chim., [3], **7**, 104.

‡ Richards, These Proceedings, **28**, 7 (1893).

Two hundred grams of the purest crystals, after having been dried at 139°, were dissolved in about a liter of the purest water and filtered into a large platinum dish, into which was passed at first pure ammonia gas and then pure carbon dioxide through a platinum tube. The pure strontic carbonate was washed by decantation eight or ten times, dried on the steam bath, and ignited in a double platinum crucible over a Berzelius lamp. Thus were residual traces of nitrate decomposed.

The mixture of oxide and carbonate obtained in this manner was dissolved in very pure hydrochloric acid, which had been distilled in a platinum still. The chloride was evaporated to dryness, fused in a platinum dish to render silica insoluble, dissolved in purest water, allowed to stand, filtered, acidified, crystallized, and dried. That used for Analysis 1 was crystallized only once, that used for Analysis 4 was crystallized three times, while that used for Analyses 2 and 3 came from the evaporated mother liquors of the last crystallization. The essential agreement of the results from these three fractions shows that if any barium had escaped separation as sulphate, it was eliminated by the first recrystallization as chloride. This is not surprising, since baric chloride is heteromorphous with the strontic salt, these substances crystallizing with two and six molecules of water respectively, at ordinary temperatures. The previous double crystallization as nitrate could not have eliminated barium, since strontic and baric nitrates cystallize in similar anhydrous forms.

The silver used was similar to that used in the work on magnesic chloride.* It was finally prepared for weighing by fusing the electrolytic crystals in a boat of lime in a vacuum. Subsequent work has shown that this silver may have contained as much as $\frac{1}{30000}$ of its weight of oxygen, derived from the decomposition of the included argentic nitrate from which the crystals were deposited; but no correction is made for this trace of impurity, because its amount is uncertain, and because the analytical work can hardly be accurate to within one part in thirty thousand.

Before analysis the strontic chloride was fused in a current of hydrochloric acid and nitrogen, cooled in pure dry nitrogen, and transferred in its platinum boat to the stoppered weighing bottle in pure dry air. For the first determination the apparatus used with the bromide was employed for these operations, but in the last three analyses the improved form devised for the work on magnesic chloride was employed.†

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* Richards and Parker, These Proceedings, **32**, 62 (1896).

† Ibid., **32**, 58 (1896).

These last three determinations were very kindly made by Dr. H. G. Parker immediately after the completion of the magnesium work, and all the precautions discussed in that description were heeded. It is a pleasure to express my indebtedness to Dr. Parker for this assistance.

The analysis of the three fractions of crystallized salt were made by comparison with silver, using solutions at least as dilute as decinormal, and determining the end point by observing with the help of the nephelometer when the supernatant mother liquor contained equal concentrations of silver and chlorine.

A preliminary experiment with salt of less purity need not be detailed. The table below contains the essential data and results of the four consecutive final determinations, the only ones which were made.

The Ratio of Strontic Chloride to Silver.

Number of Determination.	Weight of Fused Anhydrous $SrCl_2$ (in Vacuum).	Weight of Fused Silver (in Vacuum).	Ratio $2\,Ag : SrCl_2 = 100.000 : x$.
	grams.	grams.	$x =$
1	4.2516	5.7864	73.476
2	2.4019	3.2688	73.480
3	3.5181	4.7886	73.475
4	3.0264	4.1189	73.476
			73.477

If silver is assumed to have an atomic weight of 107.93, and chlorine 35.455, according to usual custom, the atomic weight of strontium calculated from these results becomes 87.697. This result, greater by 0.033 than the value found from the bromide, indicated the presence somewhere of an unknown source of error; and the results were left unpublished for ten years because of the doubt therefore attaching to them.

This doubt has now been wholly dispelled by the discovery of an error in the assumed atomic weight of chlorine. It now appears that if silver is taken as 107.93, chlorine must be 35.473;* and the atomic weight of strontium calculated upon this basis becomes 87.661, a value essentially identical with that found from the bromide, 87.663.

* Richards and Wells, in an investigation now being published by the Carnegie Institution. The somewhat lower value, 35.467, which was first announced as the outcome of this work, was the result of preliminary experiments only.

It must be borne in mind that the standard $Ag = 107.930$ is probably not precise, if oxygen is taken as 16.000. The recent work just cited has shown that the silver of Stas must have contained weighable traces of impurity, and therefore that silver must have a lower atomic weight than 107.93. Because, however, the exact value is still uncertain, the value 107.930 is still retained, in order to avoid an additional shift from one arbitrary standard to another. An alteration in the assumed value for silver affects that for strontium in direct proportion, and may therefore be applied at any time.

Summary.

In this paper are detailed four determinations of the atomic weight of strontium, by comparison of strontic chloride with silver. Calculated upon the basis of the old incorrect value for chlorine, these results yield a value higher than that obtained from strontic bromide; but on the basis of the new value for chlorine 35.473 (silver being assumed as 107.930) the two different series yield almost identical results. Accordingly the new value for chlorine is confirmed by this investigation. The atomic weight of strontium is thus found as follows ($Ag = 107.930$).

From $SrBr_2$; $Sr = \mathit{87.663}$
From $SrCl_2$; $Sr = \mathit{87.661}$
Average $Sr = \mathit{87.662}$

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VISCOSITY OF AIR.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

VISCOSITY OF AIR.

By J. L. Hogg.

Presented by John Trowbridge, December 14, 1904. Received January 9, 1905.

From the standpoint of the Kinetic Theory of Gases the viscosity, or internal friction, of a gas is a very important property, and, on this account, it is not surprising that a determination of the coefficient of viscosity of various gases has been the subject of investigation for many scientists. Air has been submitted to investigation probably more frequently than any other gas, but although many careful experiments have been performed and though the results of the more recent measurements show much better agreement than the earlier results showed, yet there is still some lack of agreement in the results obtained by different investigators using the same method as well as in those obtained by different methods. For example, O. E. Meyer * using one method obtained results ranging from 0.000168 to 0.000174 for the value of the coefficient of viscosity at 0° C. Breitenbach,† using the same method, got 0.0001733, which is in good agreement with the result obtained by Schultze ‡ and also with that obtained by Markowski. § By the same method, however, Schneebeli ‖ obtained 0.0001707 and Obermeyer ¶ 0.0001705, Tomlinson,** using a method involving another principle, obtained as a mean result 0.0001715, while F. G. Reynolds,†† by the same general method, obtained 0.0001767.

Since experiments on the relation of viscosity to temperature show a comparatively slow change in viscosity with changing temperature, and since the temperature of the gas is the only property of it which is likely to be subject to even moderate change during the course of an experiment, it would seem that it should be possible to determine the

* Pogg. Ann., cxlviii.

† Ann. der Phys., 5, 1901.

‡ Ann. der Phys., 5, 1901.

§ Ann. der Phys., 9, 1904.

‖ Arch. Sci. Phys. et Nat., 14, 1885.

¶ Wiener Sitzungsber. Abth., 2, 1875.

** Phil. Trans., 177, 1886.

†† Phys. Rev., July, 1904.

coefficient of viscosity with more precision than that indicated by the results hitherto obtained. It would appear that there may be some cause for the discrepancies other than the mere accidental errors which are present in every investigation. A further examination of the subject would, at least, seem justifiable.

Without reference to the definition of viscosity from the point of view of the Theory of Gases, it may be defined as that property of a gas by virtue of which resistance is offered to the motion of one portion of the gas relative to the contiguous portion. The coefficient of viscosity of the gas is defined as the force per unit surface which must be applied parallel to the surface of a plane placed in a mass of gas at unit distance from a fixed plane, parallel to the former, so that the moving plane may have unit velocity in the direction of the force. To measure this coefficient, which is usually denoted by μ, a shearing strain must be produced in the gas and the force necessary to do the shearing measured. The ideal shear would be that obtained by causing one plane in a mass of gas to move with a constant velocity parallel to another fixed plane; but the planes would have to be infinite to avoid the edge effects, so that this method is not practicable.

One of the two general methods of shearing which approach this ideal case is by oscillating a solid body in the gas. The resistance offered by the gas to the motion of the solid is determined by observing the diminution of the amplitude of the oscillations. The quantity actually observed is the arc of swing, and the decrement in the logarithm of this arc is then calculated. In order, then, to determine the coefficient of viscosity, there must be a mathematical expression showing the relation it bears to this logarithmic decrement. The desired relation has been obtained in the case of pendulums having the form of a sphere, an infinitely long circular cylinder, and a circular plate oscillating in its own plane between and parallel to two fixed disks. The cases of the sphere and cylinder were first discussed by Professor Stokes,* while that of the disk was treated by Maxwell. The other method of producing the desired strain, called the transpiration method, consists in causing the gas in question to flow through a tube of narrow bore, a small constant difference of pressure being maintained between the ends of the tube. O. E. Meyer † has discussed the flow of the gas in such circumstances

* Camb. Phil. Soc. Proc., **9**. Also a note to Tomlinson's paper, Phil., Trans., 1886.

† Pogg. Ann., **127**, 1866.

and has deduced an expression for μ in terms of the radius and length of the tube, the mean pressure in the gas, and the excess of the pressure at one end over that at the other.

That there are discrepancies in the results obtained is probably accounted for, as Meyer has pointed out, by the failure of the experimental methods to conform to the conditions imposed in the mathematical analysis. Whatever method is chosen, then, the prime requisite is to be certain that the mathematical conditions are fulfilled.

As has been said, Maxwell gave a theoretical discussion of the case of a circular plate performing oscillations between two fixed parallel disks and parallel to them. He used the formula obtained to get the value of μ, but his mean result is much larger than that obtained by more recent investigators using in some cases the transpiration method and in others the oscillation method. The probable cause of this was discussed by Stokes in a note appended to Tomlinson's paper on the determination of μ. Stokes showed that unless the swinging disk is adjusted so that, as it swings, it deviates very little from the horizontal plane, there will be an appreciable loss of energy, on the part of the moving disk, due to the crowding of the air between the fixed and moving disks.

Tomlinson, in the investigation referred to, by oscillating first a long cylinder and then a short one whose time of swing was the same as that of the long one, obtained, by eliminating in this way the end effects, practically a cylinder of infinite length, and was thus able to make use of Stokes's formula for the case of an infinitely long cylinder, suspended vertically and performing oscillations about its axis. There does not seem to have been any objection raised to the use of this formula for the determination of μ. His most consistent results were those obtained in this way from a single cylinder, but during the course of the investigation he made use of a system of two vertical cylinders, and of one of two spheres suspended side by side.

More recently F. G. Reynolds used a single spherical shell, and also a single cylinder, and he obtained results from the two methods which agree well with each other, but are considerably larger than the mean of Tomlinson's results, and larger than the result obtained by the latter with a single cylinder. For μ at 0° C., Tomlinson got 0.0001715, using the single cylinder, while F. G. Reynolds got 0.0001768.

The seeming completeness and simplicity of the theoretical treatment of the motion of an infinite fluid in which a solid sphere is oscillating about a vertical diameter commends for use in determining μ a pendulum spherical in form, provided it can be shown that, under the given ex-

perimental conditions, any quantities neglected in the solution of the differential equations involved are really negligible.

Professor Stokes, in his discussion of the motion of an infinite viscous fluid disturbed by a sphere oscillating about a vertical diameter, neglects, in the general equations expressing the motion, terms which are of the second degree; e. g., the square of the velocity of the fluid at any point is neglected. The assumption that these terms are negligible leads to a comparatively simple expression relating the logarithmic decrement, the density of the fluid, the inertia of the whole pendulum, the time of swing, and μ the coefficient of viscosity. But it has been shown that in the case of the sphere the terms which Stokes has neglected are not by any means always negligible.* These terms, it appears, are of the order of magnitude of $V^2 a \rho / \mu$, where V is the velocity of the fluid at any point, a the radius of the sphere, ρ the density of the fluid, and μ the coefficient of viscosity. In order, then, that Stokes's solution may be valid, the quantity $V a \rho / \mu$ must be small. V and a are the quantities which can be varied in this expression, and if they can be made sufficiently small, the determination of μ should be rendered merely a question of manipulation.

Two very good tests can be applied to determine whether the desired conditions have been obtained. In the first place, if $V^2 a \rho / \mu$ is too great to be neglected, then as V changes so does the relation between the velocity with which the solid body moves and the resistance with which it meets, and therefore the logarithmic decrement must change as the velocity changes. If, therefore, the period of vibration remains constant, the decrement measured when the pendulum is moving through large arcs should be different from that measured when the arcs are small. Thus, if the pendulum be given a certain amplitude of oscillation at the start, and the decrement be measured from arc to arc, there should be a gradual change in its value as the arc of swing diminishes. Again, when the period of oscillation, the radius of the sphere, and the angle through which the sphere moves are known, the maximum value of V can be calculated and compared with its square. Also, by using one of the values of μ, given above, the value of the quantity $V a \rho / \mu$ can be calculated.

Stokes's discussion of the sphere oscillating in an infinite viscous fluid leads to the following expression for the logarithmic decrement:

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* Whitehead, Quar. Jour. Math., **23**, 1889. Lord Rayleigh, Phil. Mag., **36** 1893.

$$\text{Nap. log. dec.} = \frac{\mu' M'}{MK^2} \cdot \left[\frac{\nu a + 3 + \dfrac{3}{\nu a} + \dfrac{3}{2\nu^2 a^2}}{1 + \dfrac{1}{\nu a} + \dfrac{1}{2\nu^2 a^2}} \right] T.$$

Where $\mu' = \frac{\mu}{\rho}$, μ is the coefficient of viscosity and ρ the density, MK^2 is the moment of inertia of the pendulum, a is the radius of the sphere, T the time of a single swing of the pendulum, M' the mass of the fluid displaced by the sphere, and

$$\nu = \sqrt{\frac{\pi}{2\mu' T}} = \sqrt{\frac{\pi\rho}{2\mu T}}.$$

The decrement here considered is, of course, that due to the resistance of the fluid on the sphere alone.

General Method.

The constancy of the elasticity of quartz fibres commended the use of one of them as a suspension. Since the sphere had to be provided with a stiff wire support to which to fasten the fibre, and on which to support a small mirror to be used in reading the arc of swing, it was necessary to devise a plan by which the friction of the air on the mirror, wire, and fibre, and also the friction in the fibre itself, might be eliminated. The following plan was used. First, a sphere was made of suitable size. The size was accurately measured, and the density of the material of which it was composed determined. Another material was then chosen whose density was two or three times that of the sphere. Two disks of the same dimensions were then made, one of the heavy material and the other of the same material as that of which the sphere was made. The dimensions of the disks were determined by two relations. First, the moment of inertia of the heavy disk about its axis had to be the sum of the moment of inertia of the sphere about a diameter and of the light disk about its axis; and second, the weight of the heavy disk had to be that of the sphere and light disk. Thus, a pendulum formed of the sphere and light disk fixed on a wire passing along the axis of the disk and a diameter of the sphere would have the same period of swing as that of a pendulum consisting of the heavy disk alone mounted as the light disk was. The tension of the supporting fibre would also be the same in the two cases. Two experiments were then performed, one to get the decrement when the vibrator consisted of the sphere and light disk, supported as described on the wire, and the other to get the decre-

ment when it consisted of the heavy disk supported on the wire at the same place as that formerly occupied by the light disk. The difference between these decrements was that due to the resistance of the air on the sphere alone.

Description of Apparatus.

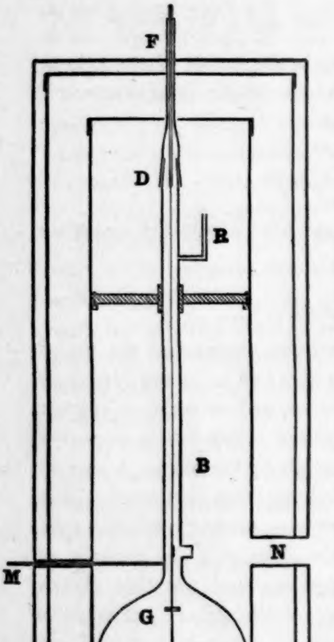

Figure 1.

Figure 1 shows the form of the apparatus which was used. A is a glass globe about 14 cm. in diameter to which, above, is connected the tube, B, and below the tube, C. The upper end of B is ground to fit the cap, D, while the lower end of C is ground to fit the mouth of the drying tube, E. The cap, D, is continued above into the tube, F, through which a brass rod surrounded by a rubber tube can just pass. To the lower end of the brass rod the fibre is attached. This method of supporting the fibre enables one to raise or lower the whole suspension very easily. The fibre is about 0.0017 cm. in diameter, and about 35 cm. in length. It is fastened at the lower end by means of shellac to an aluminum wire, G, which bears the mirror, the disk, and the sphere, H. The wire is about 12 cm. long and 0.075 cm. in diameter. The sphere, as finally used, is of glass 1.499 cm. in diameter, the variation in its diameters being less than 0.001 cm. It weighs 4.495 grams. The disk finally used is of glass. It is 1.339 cm. in diameter, 0.364 cm. in thickness, and weighs 1.297 grams. It was made with as great accuracy as the sphere. The sphere and disk are shown at Figure 2. The heavy disk is made of lead. It is of the same size as the glass disk, and weighs 5.793 grams. The mean radius of the lead disk differs only by about 0.001 cm. from that of the glass disk. The difference in thickness is less than 0.001 cm. The disk is fastened on the

wire about 5 cm. above the sphere, and is carefully fixed so that the wire is at right angles to its plane. The sphere also is adjusted so that the wire passes along a diameter. The adjustment is maintained by a small quantity of beeswax. The containing vessel, L, the top of which is removable, is a double-walled tin vessel, having an interspace of about 5 cm. This space is filled with water, while a heavy coating of hair felt surrounds the whole. An extra thickness of felt is placed on the top, as here there is no water protection. M is a thermometer. N is the glass-covered opening in the vessel, L, through which the image of a scale can be observed in a telescope by reflection from the mirror. On the top of the containing vessel is a card with its centre at the centre of the lid, and to the prolongation of the cap, D, is fastened an index which marks the zero position of D, i. e., the position in which it must be placed that the front of the mirror may be directed towards the opening, N. By giving D a slight turn a start can be given to the vibrator. A carefully constructed manometer is connected to the tube R, which is also attached to a mercury pump. The connection to the pump has a branch in which is placed a drying tube and through which, if desired, air can be admitted into the apparatus.

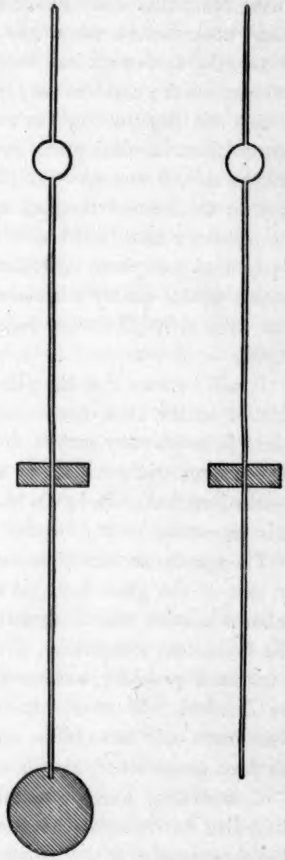

FIGURE 2.

PRELIMINARY EXPERIMENTS.

Preliminary experiments showed clearly that, in order to obtain a regular diminution in the arc of swing, great care was necessary to avoid any convection currents in the apparatus. The water jacket and felt covering described above gave entire satisfaction in this respect and also maintained the temperature of the apparatus sufficiently constant.

A test was now made to determine whether the velocity of the sphere was such as to cause a different logarithmic decrement for large arcs of swing from that obtained for small arcs. The result obtained from the many observations taken was that, for the range which it was proposed to use, the decrement was independent of the amplitude of swing. The velocity of the fluid at any point would therefore seem to have been within the limit set by the mathematical conditions. Moreover, as will appear from the data given later, the maximum angle through which the sphere turned was about 0.015 radians, the time of one complete swing was, in the case of the glass sphere, 83.52 seconds, while the radius of the sphere was 0.7495 cm. The maximum velocity of a point on the equator of the sphere must have been about 0.0011 cm. per second. The square of this quantity is about 0.1 per cent of the quantity itself. Using the value 0.00017 for μ_0 the expression $V a \rho / \mu$ must have been about 0.006.

It will be seen that the plan described above for the elimination of the friction on the fibre, mirror, and wire, as well as the friction in the fibre itself, is satisfactory only if the friction in the fibre remains constant during the experiment. Even with quartz this state of constancy is not reached until the pendulum has continued swinging for some time. With this precaution a very regular value for the decrement was obtainable.

Though the surface of the lead disk seemed to be little, if any, inferior to that of the glass disk, yet to fully justify the replacing of one by the other a wooden disk with a smooth surface was used as a pendulum and the decrement measured. Then the same disk, having its surface blackened with graphite, was used, but no difference in the decrement could be detected. Want of smoothness, merely, should, indeed, affect the decrement only in so far as a rough surface exposes more surface to the air for a given inertia than a smooth surface does.

Conceivably, also, there might be a damping of the swinging of the lead disk due to induced currents in the lead. This was tested by changing the intensity of the magnetic field surrounding the vibrator to two or three times its original value. No change in the speed of damping could be observed.

Method of Experiment.

The main part of the experiment was carried out as follows: After the sphere and disk had been fastened on the wire in the manner already described, the whole suspension was lowered into the containing vessel until the cap, D, rested in its place. When the cap, D, had been ad-

justed so that the mirror should come to rest in the proper place, and time had been given for the amplitude of the oscillation to diminish to the desired magnitude,* the points of rest of the consecutive arcs were observed. No observations were made until the arc had diminished to about 15 cm. The scale distance was 2.3 metres, so that the largest angle through which the sphere turned from the position of equilibrium was about 0.015 radians. Every point of rest was observed until the amplitude of oscillation had diminished to about 5 cm. A series of twenty consecutive arcs could thus be obtained. By turning the cap, D, a fresh start was given to the vibrator and the same process repeated, care being taken to allow any pendulous motion to subside before any observations were made. Generally, from four such series the mean decrement was obtained. Before removing the sphere from the vessel the time of swing was determined. The pressure was measured at the beginning and end of the experiment, and the temperature observed from time to time during the course of the work.

The suspended system was then taken from the vessel, the sphere and disk removed, and the lead disk fixed on the wire at the same place as that previously occupied by the other disk. When the new system had been returned to the vessel and the swinging of the disk had sufficiently subsided, the same process as was carried out with the sphere was repeated with this new pendulum. The time of swing, the pressure, and temperature were again observed.

These observations gave all the data necessary for the calculation of μ with the exception of the moment of inertia of the vibrator. To get this, three cylinders were made having different radii, and each a length sufficient that its weight should be just that of the lead disk. The weights were made the same to a milligram. Thus the fibre when supporting any one of the cylinders was under the same strain as when it was supporting the sphere and disk. The moment of inertia of each cylinder was calculated, and then a knowledge of the time of swing of each and of that of the wire and mirror when allowed to oscillate alone gave at once the value of the moment of inertia of the mirror and wire. This was added to the moment of inertia of each cylinder, and then from each result the moment of inertia of the vibrator was obtained. The moment of inertia of the vibrator could be calculated, but it seemed better to get it from the time of swing which the vibrator had in the course of the experiment.

* This process usually took between one and two hours.

Experiment I.

In the first experiment the sphere was made of ivory. Its diameter was 0.8653 cm., and together with the ivory disk weighed 5.460 grams. The moment of inertia of the mirror and wire alone was found to be 0.0016 in C. G. S. units, while that of the vibrator as determined from the different cylinders, which in this case were made of ivory, was 1.6409, 1.6424, and 1.6479, the mean of which is 1.644. The temperature corrected by a standard thermometer was 23°.0, and the pressure 75.65 cm. The logarithmic decrement obtained when the disk was on the wire was 0.00859 and the time of swing 38.83 seconds. The decrement for the sphere and ivory disk was 0.02424, and the time 38.94 seconds. To change the former decrement to what it would be were the time of swing 38.94 seconds the approximate formula $\lambda\,(1-t_2/t_1)$ was used. It expresses the amount to be added to λ, the decrement when the time is t_1 in order to get what that decrement would become when the time had changed to t_2 and is obtained by considering the pendulum as being damped by a resistance proportional to the angular velocity. It is sufficiently accurate for this correction, which must be small, since the difference in time is small. In this case the correction was —0.0002, so that the corrected decrement for the lead disk was 0.00857 and the decrement due to the resistance to the sphere alone must have been 0.01567. To the base e, this becomes 0.03608.

The following table gives the data necessary for the calculation of μ:

TABLE I.

Temp.	ρ.	T.	Dec.	MK^2.	M'.	a.
23°.0	0.0011871	38.94	0.03608	1.644	0.0032215	0.8653

The insertion of these values in the expression for the logarithmic decrement in Stokes's formula leads to an equation of the fourth degree from which to determine $\mu^{\frac{1}{2}}$. The solution gives

$$\mu_{23^\circ.0} = 0.0001825.$$

Holman * has obtained the formula

$$\mu_t = \mu_0\,(1 + .002751\,t - .00000034\,t^2)$$

* Phil. Mag., **21**, 1886.

to express the variation of μ with the temperature. This formula has been used to reduce the above result to 0° C.

$$\mu_0 = 0.0001716.$$

Experiment II.

After being in use for some time, the ivory sphere and disk were found to have changed in weight and shape, so that the ivory was replaced by glass. The dimensions of the glass sphere and the requisite disk have been given in the description of the apparatus. The method of performing the experiment was the same in this case as in the previous one. Metal cylinders, however, were used for determining the moment of inertia. The latter was found to be, from the first cylinder, 1.3047, from the second 1.3042, and from the third 1.3029. The mean is 1.3039. The decrement for the sphere and disk was 0.02211, and for the lead disk 0.00862. The time for the lead disk was 41.77 seconds, so that no correction for time was necessary. The results are summarized in Table II.: —

TABLE II.

Temp.	ρ.	T.	Dec.	MK^2.	M'.	a.
18°.6	0.001228	41.795	0.03106	1.3039	0.002166	0.7495

From the data given in this table

$$\mu_{18^\circ.6} = 0.0001795.$$

When reduced to 0° C. this gives

$$\mu_0 = 0.0001708.$$

Experiment III.

In this experiment the temperature of the gas when the sphere was suspended was 15°.6 C., while when the lead disk was suspended it was 14°.8 C. The correction to be applied was obtained by getting the decrement for the lead disk when the temperature was 15°.3 C., and using this as a basis for determining what it would be at 15°.6. In this way the correction was found to be 0.00004. This correction was not quite as satisfactory as might have been desired, as appeared when the decrement was deduced from the observations at 15°.3. The results were not

quite so regular as they were in some other cases. This was probably due to having taken observations to be used without having given sufficient training to the fibre. But there can be no serious error involved in the corrected decrement (probably not more than 0.1 per cent) as the correction is necessarily small.

The pressure in this case, as in the last, was also observed to be slightly different when the sphere was used from that when the lead disk was used. But by the same method as that used in the case of the temperature the correction was found to be negligible.

As the time for the lead disk was 41.755, no correction for time was necessary.

Table III is a summary of the data obtained from this experiment.

TABLE III.

Temp.	ρ.	T.	Dec.	MK^2.	M'.	a.
15°.6	0.0012316	41.76	0.03095	1.3039	0 0021721	0.7495

From the data given in this table

$$\mu_{15^\circ.6} = 0.0001790,$$

and

$$\mu_0 = 0.0001716.$$

The value of μ has been determined also for pressures less than atmospheric. The following tables give the results obtained : —

TABLE IV. $p = 36.034$ cm.

Temp.	ρ.	T.	Dec.	MK^2.	M'.	a.
15°.6	0.0005799	41.76	0.03049	1.3039	0.0010227	0.7495

From the data given here,

$$\mu_{15^\circ.6} = 0.0001782,$$

and

$$\mu_0 = 0.0001709.$$

TABLE V. $p = 10.014$ cm.

Temp.	ρ.	T.	Dec.	MK^2.	M'.	a.
15°.6	0.0001612	41.76	0.03037	1.3039	0.0002842	0.7495

Here, $\mu_{15^\circ.6} = 0.0001786,$

and $\mu_0 = 0.0001712.$

TABLE VI. $p = 2.499$ cm.

Temp.	ρ.	T.	Dec.	MK^2.	M'.	a.
15°.6	0.00004022	41.76	0.03035	1.3039	0.00007093	0.7495

In this case $\mu_{15^\circ.6} = 0.0001786,$

and $\mu_0 = 0.0001713.$

A reference to the description of the apparatus will show that the distance between the fixed and moving surfaces, viz., the inner surface of the glass bulb and the surface of the sphere, was about 6.5 cm., so that no correction need be applied for slip.

In addition to what has been said with regard to the neglect of the squares of velocities it may be urged that, if it can be taken as satisfactorily shown that the value of μ is constant over as large a range of pressure as that over which the foregoing experiments have been made, there is, in the results obtained at the lower pressures, another evidence of the justifiable neglect of quantities of the order of $V^2 a \rho/\mu$. For a diminution of ρ diminishes this quantity while not affecting μ, so that, if the value of μ derived from a formula obtained by neglecting the squares of small quantities is the same at atmospheric pressure as it is at a much smaller pressure, 2.5 cm., it may be safely concluded that the use of the formula at atmospheric pressure was justifiable.

The mean of the results obtained at atmospheric pressure is

$$\mu_0 = 0.0001713.$$

This is very nearly the mean of the results obtained by the more recent investigators, some of whom used the transpiration method and others the oscillation method.

Considered from the point of view of the Kinetic Theory of Gases, the viscosity of a gas is measured by the excess of the quantity of momentum transferred in one direction across the mutual boundary of two portions of the gas by the to and fro motions of the particles over the quantity of momentum transferred in the opposite direction. The ions in a gas no doubt play their part in transferring momentum from one layer of the gas to another, but their number is so exceedingly small in comparison with the number of unaffected particles that it is scarcely probable that the transfer of momentum is appreciably affected by the ionization of the gas.

An experiment was performed with the object of ascertaining whether there is any appreciable change in the viscosity of air when in the ionized state. No change in the rate of damping of the oscillation could be observed.

The investigations of Tomlinson and Reynolds, already referred to, seem to be the only ones in which the oscillating body was spherical. A closer examination of these investigations may therefore be permitted. In Tomlinson's experiment two spheres, each 6.3 cm. in diameter, were fixed side by side with their centres 20.78 cm. apart. The system oscillated about an axis, which passed along a vertical line midway between the spheres. The time of one complete swing was 5.76 seconds. The mathematical formula for this case, which was given by Stokes, required a correction for the rotation of the spheres about their respective vertical diameters. When adapted to the case of a single sphere the formula for this correction is that made use of in the present investigation. From the experiment with the spheres Tomlinson found $\mu_0 = 0.0001716$.

The results obtained by the same investigator by means of the two-cylinder method, and by means of the single-cylinder method, showed good agreement with each other and with that given for the spheres. A vibrator in the form of a single cylinder was finally adopted and the two results obtained from it were $\mu_0 = 0.0001716$ and $\mu_0 = 0.0001714$.

Where cylinders are used it would appear that there is not the same limitations for the velocity that there are when spheres are used, and consequently, from considerations of what the velocity of a point on either sphere must have been, the result given for the spheres is perhaps hardly what one might expect. With an amplitude of oscillation, such that the vibrator would move from the point of equilibrium through an angle of 0.02 radians, the maximum velocity of the centre of each sphere must have been about 0.23 cm. per second. The value of $Va\rho/\mu$ would

in this case be about 5.5. On this account one would have expected a larger result than that given.

In the experiment performed by F. G. Reynolds the spherical shell which he used had a diameter of nearly 13 cm. It was made to vibrate about a diameter, and the period of oscillation was 16 seconds. The maximum angle, measured from the position of equilibrium, was apparently about 0.15 radians, so that the maximum velocity of a point on the equator of the sphere must have been approximately 0.19 cm. per second. The square of the quantity is not small compared with the quantity itself, and the quantity $Va\rho/\mu$ for this case is about 9.4. There may be in this an explanation of the fact that the result obtained by Reynolds is about 3 per cent greater than the mean of the results given in this paper.

The analogy between the method of the single cylinder and that employed in the present paper is apparent. As has been seen, the result obtained by Tomlinson by this method is almost identical with that obtained in the present investigation; but Reynolds, by this method, obtained $\mu_0 = 0.0001768$. An effort has been made to discover the cause for this discrepancy.

In order to eliminate the effect of friction on the mirror and wire and the viscosity in the wire itself, Reynolds made use of a telescopic arrangement of two hollow cylinders. The decrement was measured, first, with the outer cylinder covering the inner, and then with the inner one pulled out a distance of 25.4 cm. The difference between these decrements he took to be that due to the friction of the air on the inside and outside of a length of 25.4 cm. of the inner cylinder. The inside diameter of the inner cylinder was 4.70 cm., and the inside diameter of the outer was 5.00 cm. The thickness of the metal was 0.04 cm., so that there was a space of 0.11 cm. between the cylinders. In obtaining in the manner described the decrement for the extended portion of the inner cylinder the assumption was made that the friction on the inner surface of a length of 25.4 cm. of the outer cylinder was the same as the friction on the same length of the inside surface of the inner cylinder together with the friction due to a length of 25.4 cm. of the layer of air between the cylinders. This assumption would seem to require justification.

On account of the two distinct operations involved in the method used in the present investigation, it is not well suited for the determination of viscosity at low pressures. Another investigation is in progress, one of whose objects is to determine the law relating viscosity to pressure at pressures where slip is appreciable and, when this is done, to measure

very low pressures by measuring viscosity. As in Maxwell's formula for the oscillating disk, account is taken of the friction on and in the fibre, and on the mirror, the sphere has been replaced by a glass disk oscillating between two fixed glass disks. The whole is enclosed in a glass vessel which is in connection with a McLeod Gauge and a mercury pump.

I desire to express my gratitude to Professor Trowbridge for placing at my disposal everything necessary for carrying on the work, and to him and to the other professors of the department for helpful suggestions during the course of the investigation.